mathNEWS

What's inside...?





mastHEAD

"WHAT'S THE BEST PART OF A mathNEWS ISSUE?"

Thanks for reading this special issue. It's been a long time in the making, so we thought we'd walk you through what it took to make it.

- Mar. 2022: Editor retreat in Las Vegas to determine content and layout
- Apr. 2022: Issues support USB-C charging
- Aug: 2022: Test issues released and hunted for sport
- **Sep. 2022**: Issues found to pose choking hazard to children under three
- Dec. 2022: Second Las Vegas retreat to win back our money
- Feb. 2023: Issues battered and fried before release

We hope you enjoy!

god ∲ peED Editor, math**NEWS** CLARIFIED | the cover! ♣ ♥ ♥

DISTRACTED | the ISSN! ♣ ♥ ♥

EVALUATED | the gridWORD! ♣ ♥ ♥

GOD ♦ PEED | the mastHEAD! ♣ ♥ ♥

UKNIGHTED | the mathNEWS Cartoons! ♣ ♥ ♥



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1. Indefinite article

DOWN

1. The letter "a"



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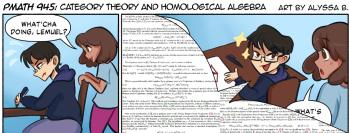
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	a p-driebb, (untountically) of finite Z _p -counts. This Z_150(W_1F)(P-modules	implies that one has a canonical isomorphism of (1.1) $EW_{k_0(1)}(K_{2m-1}(O_{k(1)})^{m-1})W_k(1-$	
	OSS ACCUSATION OSS	COLYY K(OL) denotes the i-th Quillen K-group	p) assumption for (I_n) but not significant for (I_n) . Show, The main paid of the ring $C_{H,n}$ of S integers in M and $v_n = 1/2(1+(-1)^n)$. (I_n) on S and T_n and see the
	beautiche de la commission de La	of the Data models of MCCR, in The county December 1.8, in 197, the confuse series as 2	NC for the Pennson models $\mathbb{Z}_2(M_1^*(K_n)^*)$, but taking has, in the spirit of [1] for the Schr
	This result is the sharing point for the definition belowing from Vencon part and	as affine Dirac scheme S , the co-entegrey $QCob(S)_{cos}$ is Crothendieck prostable in	
	The result is the starting point for the destination indicate from viscosis [14] and Lucie [25] of the property of a map of Dirac stacks $f, Y \to X$ of being geometric. Informally, the geometric maps span the smallest full subestingary	the sense of $[27]$, Definition C.I.4.2]. Thus, the symmetric monoidal adjunction	hyperplane if its have change along any map $\eta \colon S \to X$ from an affine Dirac scheme in equivalent to a formal affine space. This property is stable under base change.
$\mathcal{P}(Aff_n) \xrightarrow{t_n} \mathcal{P}(Aff_{n'})$	$\operatorname{Sin}(AE)_{fX}^{max} \subset \operatorname{Sin}(AE)_{fX}$	$QCab(S)_{10} \xrightarrow{S^{\infty}} QCab(S)$	but it does not descend along effective spin-orphisms in general, even if X is a Dirac scheme. However, if we fix a pertian, then it does. We consider a bound affine many
preserves absents with respect to the flat topology.	that contains the maps $f \colon Y \to X$ that, locally on X , are equivalent to maps of Dirac arbonics, and that is closed under the formation of the cometric regimeter	is the connective part of a ristructure, and we obtain (Q2) by right Kan extension.	to be pointed by the zero section.
As we have already mentioned, an important consequence of Theorem 1.2 is the existence of a left cuart "sheafification" functor	of groupoids with the face maps. However, the formal definition, which we give in Section 2.3 below is more complicated, because the notions of being geometric and	is the connection part of a fortierization, and use obtain $(q\beta)$ by $n(\beta)$ in an extension. More concerving, if X is a Dirac stock, then $S \in QCob(X)$ is connective if and only if $n'(S) \in QCob(S)$ is connective for every map $\eta \colon S \to X$ with S affine. It is also true that if $n'(S) \in QCob(S)$ is connective for every map $\eta \colon S \to X$ with S	
2(M) = 1 Str(M),	being flat must be defined recursively together. The property of being geometric is well-behaved: It is preserved under composition and base-change; it local on the	time that if $\psi(Y) \in QCob(X)$ is commented to every map $\psi: X \to X$ with X salline, then so is $X \in QCob(X)$, but the consense is false. In particular, the tensor mat U_X belongs to the heart of the Astructure.	·/ \
left advice to the executed includes feature Indeed, if we reduce Aff for the	target; and maps in $\text{thr}(A\Pi)_{A}^{\text{three}}$ are accountically geometric. We now explain the theory of coherent cohemology of Dirac stacks, which we		X X X
essentially small Aff_, then Lurie has proved in [19, Proposition 6.2.2.7] that a left		the commentive part is next to impossible to understand. However, for geometric Dirac studie, including Dirac subsesses, the situation improves	is a pointed formal hyperplane if its base change along any map $\eta\colon S\to X$ from an
exact shealifestion functor exists, and Theorem 1.3 and [17, Tag 02F1] show that these left adjoint functors assemble to vive the stated left adjoint functor.	(Q4) A functor QCoh: Shr(Aff)* → CAlg(LFr); it assigns to map of Dirac stacks	Proposition 1.0 (Cometric study and Estructure), W.X. is a proposition Discussion.	affine Dirac scheme is equivalent to a pointed formal affine space.
We can now deduce from \$10, Theorem 6.1.0.68 that the co-category of Direct	$f: Y \to X$ a symmetric monoidal adjunction	Proposition 1.0 (Commette stacks and Cotructure). If X is a promote Described then the Latencius on CCoh(X) is side consider and the econometric next	Filtr a suppression are 11. Marrier 1 10%
stacks satisfies the co-entegrated Circuit actions, except that it not presentable.	QCa(X)	is closed under filtered columns. Moreover, if $q \in S \rightarrow X$ is relaterator with S affine,	
Theorem 1.5 (Growt's actions for Dirac stucks). The oc-category Shr(AE) aj	qua(x) (qua(r)	then $F \in QCch(X)$ is constant time if and only if $\eta^*(F) \in QCch(S)$ is an	is the full subestagory of the so-category of pointed Dirac stacks
Dirac stacks has the following properties:	between presentably symmetric monoidal stable so entegories.	We let $\pi_i^0(\mathcal{I}) \in QCob(X)^{ij}$ be the ith homotopy \mathcal{O}_X -module of $\mathcal{I} \in QCob(X)$	$\operatorname{Hyp}_{\bullet}(X) \subset (\operatorname{Shv}(\operatorname{Aff})_{(A})_{\bullet}$
(i) It is complete and exemplete, and it is generated under small colimits by the executive image of the Youda embedding h: All → Shr(All), which consists of so-compare delete and is convenible.		with respect to the f-structure. We refer to i as the "minimized" degree in order to repensic it from the "spin" degree intrinsic to Dirac geometry, if $f\colon Y\to X$ is a	
(ii) Calculu in Shr(Aff) are universal.	$QCab(X)_{\geq 0} QCab(X)$	map of Dirac stacks, then the "cornect" relative coherent cohomology θ_X -modules of the θ_Y -module $S \in QCoh(Y)$ are given by	
(iii) Charadactic in Shy (AE) any district.	such that i is the fully faithful inclusion of the connective part of a f-structure.	No. (C. (C.) (C.) (C.) (C.) (C.) (C.) (C.)	$\mathbb{R}_{\mathrm{PP}_{\bullet}}(X) \xrightarrow{f^{*}} \mathbb{R}_{\mathrm{PP}_{\bullet}}(X^{*})$
(iv) Every prospeid in Str(Aff) is effective. The re-category of Dirac stacks in close enough to being an re-topes that it	and such that for every map of Dirac stacks $f \colon Y \to X$, the functor f^* is sight t-max, or suspendently, the functor f_* is left t-max.	**The breakly booker Chief (No - Life - Chie answer and refer broke	
retains the important property of no topoi that the contravariant functor		"The legible busines Chig(176) is 17e in Cata, present and reflect limits. This functor can recornily not be enjoylated as the derived functor of the left exact.	$(Shr(Aff)_{(X)})_{*} \xrightarrow{f^{*}} (Shr(Aff)_{(X)})_{*}$
SetMin Gi-	scheme S is $Spec(A)$ unique the presentably symmetric monoidal entegory	functor between abelian entegories for induced by the left t-exact functor for	is a extenso diagram of ex-entegrates. So by Theorem 1.5, the ex-entegray of
that to X assigns the slice co-category Nov(Aff) y takes colimits of Dirac stacks	QCub(S) ²⁰ to Mod ₄ (Mb)	Another special class of Dirac stacks that we consider are the formal Dirac schemes, if $n : S \to X$ is a closed immersion of Dirac schemes, where defining	
to limits of co-entegrates. This statement is the source of all descent statements in this coper. Typically, we wish to show that the full subcategory	of graded A-modules. It promotes via animation to a functor that to S assigns the promoted essentially commercial essential co-category	quasi-coherent ideal $I \subset \mathcal{O}_A$ is of finite type, then we define the formal completion of X along S to be the collect in Dirac stacks	Theorem 1.9 (First decoun for pointed formal hyperplanes). If $f\colon X'\to X$ is an effective epimorphism of Dirac abods, then the conomical maps
$Shv(Alf)_{J,X}^{p} \subset Shv(Alf)_{J,X}$	$\mathrm{QCLh}(S)_{\geq 0}\simeq \mathrm{Mod}_{\mathfrak{A}}(Ab)^{\mathrm{ort}}$	$Y = \lim_{i \to \infty} S^{(m)} \xrightarrow{i} X$	$\operatorname{Hyp}_{\mathbf{v}}(X) \longrightarrow \bigoplus_{\mathbf{i} \in \mathbb{N}} \psi_{\mathbf{i}}(X^{-\mathbf{v},\mathbf{v}}(\mathbf{i})) \longrightarrow \bigoplus_{\mathbf{i} \in \{0,1,2\}} \operatorname{Hyp}_{\mathbf{v}}(X^{-\mathbf{v},\mathbf{v}}(\mathbf{i}))$
spanned by the maps $f \colon Y \to X$ that have some property P satisfies that descent. But this follows from the fundamental descent statement, every we recent that the	Territoria	of the infinitesimal thirdenines $e^{(n)} : S^{(n)} \to X$. The map it is not reconstric, but	are optimizates of co-categories.
But this follows from the fundamental descent statement, once we prove that the property P satisfies descent for the flat topology in the case that for every effective epimenthism $F: X' \rightarrow X$, the diagram of ∞ -naturation	as de questions by equivalence relations in 1 sategory theory; see [10, fection 6.1.2], of animuted conded A-modules, which, in turn, promoters via stabilization to a	if the closed immercion $\eta \colon S \to X$ is regular in the sense that, locally on X , it is defined by a recular sequence, then it behaves as an open and affine immercion of	We recall from $[17,$ Proposition A.1] that the right-hand map in Theorem 1.9 is an equivalence, because the ∞ category of pointed formal hyperplanes is equivalent
$\operatorname{Shr}(AF)_{-r}^{r} \longrightarrow \operatorname{Shr}(AF)_{-r}^{r}$		a tubular neighborhood of S in X , as excisioned by Grothendieck.	to a 1-entegory, and that the right-hand term is equivalent to the 1-entegory of resisted formal homorophism over X' with descent data above $f: X' \to X$.
SOUTH TITLE SOUTH THE	$QCub(S) \simeq Sp(MuL_{\phi}(Ab)^{**})$	Theorem 1.7 (Formal completion and recollement). Let X be a Dirac scheme,	
	of spectra is animated graded A-modules. Following Gesthendisck, we show that	and let $n \colon S \to X$ be a regular closed instancesion. Let $j \colon Y \to X$ be the formal completion of X along S , and let $i \colon U \simeq X \smallsetminus S \to X$ be the inclusion of the open	let Lat be the category of finitely penerated free abelian groups and define the
$\operatorname{Shr}(\widehat{AF})_{/X} \longrightarrow \operatorname{Shr}(\widehat{AF})_{/X}$	this functor is a sheaf for the flat topology on Alf, a fact, which, in homotopy theory, was first observed and exploited by Hopkins [14]. This gives us (Q1):	complement of S. In this situation, there is a stable recofferent	AbiC = Pin*Let*, C : PinLet*, C
is cortexion. For example, we define a functor that to a Dirac stack X assigns the	Theorem 1.3 (Sabblely the decent for coast schools mobile). The right Kon	Á Á	spanned by the functions that personnel finite products.
so entegory $\mathbb{P}Group(X)$ of formal groups over X and use this strategy to show that it descends along effective epimorphisms.	extension of the functor QCub: Aff ⁴⁰ CAlg(LPs) along the Yourds embedding to Aff TOM) admits an executivity unique factorization.	QCA(I) 00A(X) 00A(X)	Definition 1.10. The co-category of formal groups over a Dirac stack X is the co-category PGroup(X) \cong $AbtByp(X)$ of $abching group-abjects in the co-category$
It follows from Contember's faithfully flat descrit for graded modules, which we proved in IC. Theorem L.S. that the flat topology on Aff is subcassocial. Here		and, in addition, the functor j _s in t-exect.	so-conseque Pursup(A) is $Au(Ryp(A))$ of about group report in the so-category of formal hyperplanes over X .
we proved in [12] Theorem 1.3], that the list topology on All is submissional. Here, we prove that, more generally, the enterprey of Dirac schemes, which we defined in [13], Definition 2.29], enabeds fully faithfully into the on-enterprey of Dirac stacks.	through the sheeffication functor.	The formal schemes that we will enough are the formal affine spaces, defined as follows. If S is a Dirac scheme and $L \in QCA(S)^{N}$ on C_{J-ann} their levelly free of	The co-category $PGroup(X)$ is equindent to a 1-category. Moreover, if S is a formal group over X, then the underlying hyperphase SCD is pointed by the zero
Theorem L4 (belows we stock). Dirac schemes are Dirac stocks:	To spell out this definition of $QCob(X)$, if we choose a regular cardinal κ such that $X\in Shr(M)$ is the left Kan extension of $X_k\in Shr(M_k)$, then	finite rank, then the affine space associated with \mathcal{E} is the affine map $A_{\mathcal{E}}(\mathcal{E}) \approx \operatorname{Spec}(\operatorname{Space}_{\mathcal{E}}(\mathcal{E})) \xrightarrow{P} S.$	section $S(0) \to S(Z)$. Therefore, by Theorem 1.0, if $f : X' \to X$ is an effective spinnophism of Dirac stacks, then the canonical map
 If X is a Dirac scheme, then the functor h(X): Aff⁽⁰⁾ → E that to an affine orderer S majora Mart S, X) is accomable and a shad for the flat transfer. 	$QCab(X) \simeq \lim_{x \to -\infty} QCab(X)$,		$PGrosp(X) \longrightarrow g_{W_{Arrive}} PGrosp(X^{re_{A}/4})$
(2) The resulting functor h: Seh → Shr(AE) is fully fait[ful.	where the Book is indexed by (ESE,) ,) ** and is entrolated in CAle(LPV).*	and the formal affine space associated with ℓ is its formal completion	is an emiralese of co-rabouries, and the larget co-raboury is emirabed to the
			a sa representa di scriptoriale, son tar saggio scriptori il supresenti di tar

mathNEWS Cartoons

"Oreo toothpaste — that ain't right."